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On Modal-Asymptotic Analysis to Prestressed Thick Beam on Bi-Parametric Foundation Subjected to Moving Loads

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ABSTRACT

In this paper, the dynamic response of prestressed thick beam subjected to moving loads using modal-asymptotic analysis (MAA) is investigated. The main objective of this work is to obtain an analytical closed-form solution to this class of dynamical problems. To use MAA, accurate information is needed on the natural frequencies, mode shapes, orthogonality of the mode shapes, and Stubble's Asymptotic technique a prior. A thorough literature survey reveals that the method has not been reported in the existing literature, even for simple Timoshenko beams. Thus, we present complete information on how to use the MAA to derive the forced vibration responses of a simply supported Timoshenko beam subjected to moving loads. The effects of prestress, foundation parameters, and moving velocity on the dynamic characteristics of the beams are studied and described in detail. To validate the accuracy of this method, we compare the frequency parameter with the existing literature which is shown to compare favorably.

KEYWORDS: Modal-Asymptotic; Prestress; Bi-parametric foundation; moving loads; Timoshenko beam.

1. Introduction

Forced vibration of elastic bodies (stretched string, spring-mass system, rods, etc.) has been extensively studied by several authors (Prescott and Inglis, 1934; Muchnikov,1953; Kenny, 1954; Stanisic, 1968; Sadiku and Leipholz, 1989; Awodola and Omolofe, 2018) in many fields, from structural to mechanical to aerospace engineering for more than a decade. Aerospace engineers must understand dynamics to stimulate space vehicles and airplanes, while mechanical engineers must understand dynamics to isolate or control the vibration of machinery. In civil engineering and structural engineering, an understanding of structural dynamics is important in the design and retrofit of structures to withstand severe dynamic loading from environmental forces like earthquakes, strong wind, hurricanes, or moving loads like cars and pedestrians in the case of bridges. With the persistent development of science and technology, extensive application of high-performance materials, increasingly enlargement of the bridge span, and continuous increase in train speed and vehicle load, the problem of bridge vibration becomes more prominent. So far and during these years, many researchers have conducted different studies in this field. When moving loads are applied to a structure, dynamic deflections and stresses may become considerably higher than those induced by static loads. For this reason, various structures subjected to moving loads have been invested. Examples of moving loads include trains, cars, trucks, cranes, and pedestrians walking or running across bridges. Structural systems on the other hand are usually modeled as beams and plates. These may be elastic, inelastic, or viscoelastic. A simple example of these structural systems/members are bridges, railways, rail, decking slabs, elevated roadways to moving vehicles, girders, belt-drive (carrying machine chains), and even floppy disk/cassette player heads carrying tape. It is remarked at this juncture that while stationary loads or subsystems produce stress and deformation that are constant, traveling loads produce effects that are variable functions of the position of the load (which is also a function of time). Thus, when structural members are under the passage of moving loads, the interaction between the passing load and the structure makes the dynamic response analysis very complex. Under the relevance in the analysis and design of railway tracks, bridges, elevated roadways, decking slabs, etc., the dynamic response of structural members under the passage of moving loads had been extensively investigated and several experimental and numerical studies have been reported in the literature in recent years (Muchnikov, 1953; Kenny,1954; Stanisic, *et al.*, 1968; Sadiku and Leipholz, 1989; Oni and Omolofe, 2010, Jimoh *et al.*, 2017; Jimoh, 2017). In this study, the concern is beam-type flexure under moving loads. Many researchers have developed various solution techniques to the transverse vibration of Timoshenko beam which includes semi-analytical method (Esmailzadeh and Ghorashi, 1995), transform matrix method (Ashour and Farag, 2000), integral transform method (Milomir *et al.*1969), Galerkin's methods (Stanisic *et al.* 1974; Jimoh and Awelewa, 2017), finite element methods (Lou *et al.* 2006; Awodola *et al.* 2019) time-domain spectral element method (Mukherjee, *et al.* 2021), finite difference method (Esmailzadeh and Ghorashi, 1997)

The analytical solution closed-form solution for a moving load problem using MAA can be obtained when the information regarding natural frequencies, mode shapes, and the orthogonality properties of the mode shapes are derived. Many researchers have developed general solutions for the transverse vibrations of a Timoshenko beam. This includes (Han *et al.*, 1999), the general solution is obtained for two frequency range $\omega < \omega_c$ and $\omega > \omega_c$, excluding the cutoff frequency ω_c. (Kim *et al.*, 2017) and (Rensburg and Merwe, 24) developed a general solution that includes the three frequencies range i.e., $\omega \leq \omega_c$ and $\omega \geq \omega_c$ including the cutoff frequency ω_c.

However, their method of solution cannot handle prestressed Timoshenko beam resting on a bi-parametric foundation. Thus, in this study, we discuss the mathematical formulation of the general solutions of simply supported Timoshenko beam resting on a bi-parametric foundation subjected to moving loads considering the frequency $\omega \leq \omega_c$ and $\omega > \omega_c$ using MAA.

2 Problem Formulations

The The problem of prestressed Timoshenko beam of length L on bi-parametric foundation subjected to moving loads is governed by an initial boundary value system of equations. This system of equations can be written in matrix form as

$$M\frac{\partial^2 v(x,t)}{\partial t^2} + Kv(x,t) = Q(x,t) \tag{1}$$

where

$$v(x,t) = \begin{Bmatrix} u(x,t) \\ \phi(x,t) \end{Bmatrix}; \quad Q(x,t) = \begin{Bmatrix} f(x,t) \\ 0 \end{Bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad K = \begin{bmatrix} K_1 - (\Delta_1 + K_N + K_2)\frac{\partial^2}{\partial x^2} & \Delta_1 \frac{\partial}{\partial x} \\ -\Delta_2 \frac{\partial}{\partial x} & \Delta_2 - e_o \frac{\partial^2}{\partial x^2} \end{bmatrix}$$

$$\Delta_1 = \frac{\varphi G}{\rho}; \quad \Delta_2 = \frac{\varphi GA}{\rho I}; \quad K_1 = \frac{K_w}{\rho A}; \quad K_2 = \frac{K_g}{\rho A};$$

$$K_N = \frac{N_o}{\rho A}; \; f(x,t) = \frac{F(x,t)}{\rho A}; \quad e_o = \frac{EI}{\rho I} \tag{2}$$

$u(x,t)$ is the transverse displacement, $\phi(x,t)$ is the rotation of the cross section due to bending, $F(x,t)$ is the external transverse force, N_o (N) is the axial force, K_w (N/m^2) is the Winkler foundation, K_g (N) is the stiffness of the shear layer, E (N/m^2) is the young modulus, G (N/m^2) is the shear modulus, ρ (kg/m^3) is the mass density, A (m^2) is the cross-sectional area, I (m^4) is the moment of inertia, and φ is the shear correction factor. The natural and geometric boundary conditions relevant to eq. (1) are given by

$$T_s(0,t) = -T_{s1}(t) \quad or \quad u(0,t) = u_1(t);$$
$$T_s(L,t) = T_{s2}(t) \quad or \quad u(L,t) = u_2(t);$$

$$M(0,t) = \phi_1(t) \quad or \quad M(0,t) = \phi_1(t);$$
$$M(L,t) = \phi_2(t) \quad or \quad M(L,t) = \phi_2(t); \tag{3}$$

where $T_s(x,t)$ and $\phi(x,t)$ are the transverse shear force and bending moment, respectively, given as

$$T_s(x,t) = \varphi GA(\frac{\partial u}{\partial x} - \phi); \quad M(x,t) = EI\frac{\partial \phi}{\partial x}; \tag{4}$$

and the initial conditions are given as

$$v(x,0) = g(x); \quad \frac{\partial v(x,0)}{\partial t} = h(x); \tag{5}$$

3 The Free and Forced Vibration

3.1 General Solution

In order to obtain the eigenfunctions (natural modes) for the model under discussion, we must first obtain the general solutions for the free vibration problem. Thus, we consider the homogeneous equation of eq.(1) as follow

$$M\frac{\partial^2 v(x,t)}{\partial t^2} + Kv(x,t) = 0 \tag{6}$$

We assume the the solution of eq. (6) are in the following form:

$$v(x,t) = \begin{Bmatrix} U(x) \\ \Phi(x) \end{Bmatrix} e^{i\omega t} = V(x)e^{i\omega t} \quad where \quad V(x) = de^{rx} \tag{7}$$

where i is an imaginary number, ω is the angular frequency and r denotes the wave number. Therefore, substituting eq. (7) into eq. (6) yields the following algebraic equations

$$\begin{bmatrix} \beta_3 - \beta_1 r^2 & r\Delta_1 \\ r\Delta_2 & \beta_2 - e_o r^2 \end{bmatrix} de^{rx} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{8}$$

where

$$\beta_1 = (\Delta_1 + K_N + K_2); \; \beta_2 = \Delta_2 - \omega^2; \; \beta_3 = K_1 - \omega^2 \tag{9}$$

From which we obtain eigenvalues r. For the existence of non-trivial solutions, the determinant of the 2×2 matrix in eq. (8) must vanish at certain values of r. Thus, a dispersion equation is obtained as follows:

$$r^4 - \frac{1}{e_o\beta_1}(\beta_1\beta_2 + e_o\beta_3 - \Delta_1\Delta_2)r^2 + \frac{\beta_2\beta_3}{e_o\beta_1} = 0 \tag{10}$$

The eigenvalues are given by

$$r_i = \pm\sqrt{\frac{1}{2e_o\beta_1}[(\beta_1\beta_2 + e_o\beta_3 - \Delta_1\Delta_2) \pm \emptyset}$$
$$\emptyset = \sqrt{(\beta_1\beta_2 + e_o\beta_3 - \Delta_1\Delta_2)^2 - 4e_o\beta_1\beta_2\beta_3}]$$
$$for \quad i = 1,2,3,4 \tag{11}$$

The corresponding eigenvectors v_i are given by

$$v_i = \begin{bmatrix} r\Delta_2 \\ \beta_3 - \beta_1 r^2 \end{bmatrix} \quad or \quad \begin{bmatrix} \beta_2 - e_o r^2 \\ r\Delta_1 \end{bmatrix} \qquad (12)$$

Of the four roots, the two given by

$$r_j = \pm\sqrt{\frac{1}{2e_o\beta_1}[(\beta_1\beta_2 + e_o\beta_3 - \Delta_1\Delta_2) + \emptyset}$$
$$\emptyset = \sqrt{(\beta_1\beta_2 + e_o\beta_3 - \Delta_1\Delta_2)^2 - 4e_o\beta_1\beta_2\beta_3}]$$
$$for \quad j = 1,2 \qquad (13)$$

are either real or imaginary depending on the frequency ω (for a given material and geometry), and the other two roots given by

$$r_j = \pm\sqrt{\frac{1}{2e_o\beta_1}[(\beta_1\beta_2 + e_o\beta_3 - \Delta_1\Delta_2) - \emptyset}$$
$$\emptyset = \sqrt{(\beta_1\beta_2 + e_o\beta_3 - \Delta_1\Delta_2)^2 - 4e_o\beta_1\beta_2\beta_3}]$$
$$for \quad j = 3,4 \qquad (14)$$

are always imaginary. r_j are real when the frequency ω is less $\sqrt{\Delta_2}$ and are imaginary when the frequency is greater than $\sqrt{\Delta_2}$. We call this cutoff frequency or the critical frequency ω_c. Therefore, we must consider two cases when obtaining spatial solutions: (i.e $\omega \le \omega_c$ and $\omega > \omega_c$). By using the four eigenvalues given by eq. (11), the spatial solution can be written as follows:

(a) When $0 < \omega \le \omega_c$

$$V(x) \equiv \begin{Bmatrix} U(x) \\ \Phi(x) \end{Bmatrix} = \sum_{i=1}^{4} d_i v_i e^{r_i x} = d_1 u_1 e^{bx} + d_2 u_2 e^{-bx}$$
$$+ d_3 u_3 e^{iax} + d_4 u_4 e^{-iax} \qquad (15)$$

(b) When $\omega \ge \omega_c$

$$V(x) \equiv \begin{Bmatrix} U(x) \\ \Phi(x) \end{Bmatrix} = \sum_{i=1}^{4} \bar{d}_i v_i e^{r_i x} = d_1 u_1 e^{\bar{b}x} + d_2 u_2 e^{-\bar{b}x}$$
$$+ d_3 u_3 e^{iax} + d_4 u_4 e^{-iax} \qquad (16)$$

where

$$a = \sqrt{\frac{1}{2e_o\beta_1}[(\Delta_1\Delta_2 - \beta_1\beta_2 - e_o\beta_3) + \vartheta]}$$
$$\vartheta = \sqrt{(\Delta_1\Delta_2 - \beta_1\beta_2 - e_o\beta_3)^2 - 4e_o\beta_1\beta_2\beta_3} \qquad (17)$$

$$b = \sqrt{\frac{1}{2e_o\beta_1}[-(\Delta_1\Delta_2 - \beta_1\beta_2 - e_o\beta_3) + \vartheta]} \qquad (18)$$

$$\bar{b} = \sqrt{\frac{1}{2e_o\beta_1}[(\Delta_1\Delta_2 - \beta_1\beta_2 - e_o\beta_3) - \vartheta]} \qquad (19)$$

By using the results, (15) and (16) can be written in terms of sinusoidal and hyperbolic function functions as follows:

(a) When $0 < \omega \le \omega_c$

$$\begin{Bmatrix} U(x) \\ \Phi(x) \end{Bmatrix} = A_1 \begin{Bmatrix} sin(ax) \\ g_a cos(ax) \end{Bmatrix} + A_2 \begin{Bmatrix} cos(ax) \\ -g_a sin(ax) \end{Bmatrix}$$
$$+ A_3 \begin{Bmatrix} sinh(bx) \\ g_b cosh(bx) \end{Bmatrix} + A_4 \begin{Bmatrix} cosh(bx) \\ g_b sin(bx) \end{Bmatrix} \qquad (20)$$

(b) When $\omega > \omega_c$

$$\begin{Bmatrix} U(x) \\ \Phi(x) \end{Bmatrix} = A_1 \begin{Bmatrix} sin(ax) \\ g_a cos(ax) \end{Bmatrix} + A_2 \begin{Bmatrix} cos(ax) \\ -g_a sin(ax) \end{Bmatrix}$$
$$+ A_3 \begin{Bmatrix} sinh(\bar{b}x) \\ g_b cosh(\bar{b}x) \end{Bmatrix} + A_4 \begin{Bmatrix} cosh(\bar{b}x) \\ -g_b sin(\bar{b}x) \end{Bmatrix} \qquad (21)$$

Where

$$g_a = \frac{1}{a\Delta_1}(\beta_1 a^2 + K_1 - \omega^2); \; g_b = \frac{1}{b\Delta_1}(\beta_1 b^2 - K_1 + \omega^2);$$
$$g_{\bar{b}} = \frac{1}{\bar{b}\Delta_1}(\beta_1 \bar{b}^2 - K_1 + \omega^2) \qquad (22)$$

The present spatial solutions (20) and (21) are now the expression for three frequency ranges $0 < \omega \le \omega_c$ and $\omega > \omega_c$

3.2 Natural Frequency and Mode Shape

Analytical closed forms of natural frequencies and mode shapes are to obtained for specific boundary condition, our present study is limited to simply supported conditions given by

$$u(0,t) = u(L,t) = 0; \qquad EI\frac{\partial\phi(0,t)}{\partial x} = EI\frac{\partial\phi(L,t)}{\partial x} = 0 \qquad (23)$$

We shall considered three frequency ranges separately as follows:

3.2.1 When $0 < \omega < \omega_c$

By substituting eq. (20) into (23) yields a matrix equation

$$\begin{Bmatrix} U(0) \\ \Phi'(0) \\ U(L) \\ \Phi'(L) \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & -ag_a & 0 & bg_b \\ sin(aL) & cos(aL) & \sinh(bL) & \cosh(bL) \\ -ag_a sin(aL) & -ag_a cos(aL) & bg_b\sinh(bL) & bg_b\cosh(bL) \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (24)$$

For the existence of a non-trivial solution of A_i, (i = 1, 2, 3, 4) in (24), the determinant of the matrix of eigenvalue problem must vanish. Thus, we obtain

$$A_2 = A_4 = 0 \quad and \quad (ag_a + bg_b)\sin(aL)\sinh(bL) = 0 \qquad (25)$$

Since $(ag_a + bg_b) \neq 0$ and $\sinh(bL) \neq 0$, then if

$0 < \omega < \omega_c$ we have

$$\sin(a_n L) = 0 \qquad (26)$$

From which we obtain

$$a_n = \frac{n\pi}{L} \qquad (n = 1,2,3,\ldots,n_a) \qquad (27)$$

Therefore, substituting (27) into (17)–(19) yields natural frequencies ω_n given as:

$$\omega_{a(n)} = \sqrt{Z(n) - \sqrt{Z(n)^2 - 4R(n)}} \quad (n = 1,2,3,\ldots,n_a) \qquad (28)$$

where

$$Z(n) = [(\Delta_2 + K_1) - (\frac{n\pi}{L})^2(e_o + \beta_1)]$$

$$R(n) = [(\frac{n\pi}{L})^2[e_o\beta_1(\frac{n\pi}{L})^2 + \Delta_1\Delta_2] + \Delta_2(K_1 - \beta_1) - e_oK_1] \qquad (29)$$

Next we obtain the mode shapes corresponding to the natural frequencies $\omega_a(n)$ $(n = 1,2,\ldots,n_a)$ by determine the values of A_1 and A_3 from (24) in following forms:

$$A_1(n) \neq 0; \quad A_3(n) = 0 \qquad (30)$$

Thus, the nth mode shape corresponding to $\omega_a(n)$ from (20) is written as

$$V_{a(n)}(x) \equiv \begin{Bmatrix} U(x) \\ \Phi(x) \end{Bmatrix} = A_{a(n)} \begin{Bmatrix} \sin\frac{n\pi x}{L} \\ g_{a(n)}\cos\frac{n\pi x}{L} \end{Bmatrix} \quad (n = 1,2,\ldots,n_a) \qquad (31)$$

where

$$g_{a(n)} = \frac{L}{n\pi\Delta_1}[\beta_1(\frac{n\pi}{L})^2 + K_1 - \omega^2] \qquad (32)$$

3.2.2 When $\omega = \omega_c$

The general solution at $\omega = \omega_c$ can be readily obtained using equation (18) by allowing ω to approach ω_c i.e when $b = 0$. Therefore, substituting $b = 0$ and the use of L'Hospital's rule, eq. (20) becomes

$$\begin{Bmatrix} U(x) \\ \Phi(x) \end{Bmatrix} = A_1\begin{Bmatrix} sin(a_c x) \\ g_a cos(a_c x) \end{Bmatrix} + A_2\begin{Bmatrix} cos(a_c x) \\ -g_{a_c} sin(a_c x) \end{Bmatrix} + A_3\begin{Bmatrix} 0 \\ \beta_1^o \end{Bmatrix} + A_4\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \qquad (33)$$

where

$\beta_1^o = \frac{\beta_1}{\Delta_1}$; $a_c = a(at\ \ \omega = \omega_c) = \frac{n_c\pi}{L} = \sqrt{\frac{1}{e_o\beta_1}[(e_o\beta_3 - \Delta_1\Delta_2)]}$
and
$g_{a_c} = g_a(at\ \ \omega = \omega_c) = \frac{L}{n\pi\Delta_1}[\beta_1(\frac{n\pi}{L})^2 + K_1 - \Delta_2]$

(34)

By applying simply supported boundary condition given (23) yields the following eigenvalue equations

$$\begin{Bmatrix} U(0) \\ \Phi'(0) \\ U(L) \\ \Phi'(L) \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & -a_c g_{a_c} & 0 & 0 \\ \sin(a_c L) & \cos(a_c L) & 0 & 1 \\ -a_c g_{a_c}\sin(a_c L) & -a_c g_{a_c}\cos(a_c L) & 0 & 0 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (35)$$

For the existence of a non-trivial solution of A_i, (i = 1, 2, 3, 4) in (35), the determinant of the matrix of eigenvalue problem must vanish. Thus, we conclude that the cutoff frequency ω_c is also a natural frequency of a simply supported Timoshenko beam. Therefore, we must determine the mode shape corresponding to the natural frequency ω_c. We can see from (34) that $a_c > 0$ and $g_{a_c} > 0$. Therefore, from (35), it is easily shown that the following should be satisfied:

$$A_2 = A_4 = 0; \quad A_1\sin(a_c L) = 0 \quad or \quad A_1\sin(n_c\pi) = 0 \quad where \quad a_c = \frac{n_c\pi}{L} \quad (36)$$

Two following two cases shall be considered to satisfy (36):

Case 1. $A_1 = 0$, (if n_c is not an integer). In this case, the corresponding mode shape can be derived directly from (33) as follows:

$$V_{a(0)}(x) \equiv \begin{Bmatrix} U_{a(0)}(x) \\ \Phi_{a(0)}(x) \end{Bmatrix} = A_{a(0)} \begin{Bmatrix} 0 \\ \beta_1^o \end{Bmatrix} \quad (37)$$

Case 2. $A_1 \neq 0$, (if n_c is an integer). In this case, the natural frequency ω_c is equal to the natural frequency $\omega_{a(n_a)}$ of a bending mode shape and they become double frequencies given as:

$$V_{a(n_a)}(x) \equiv \begin{Bmatrix} U(x) \\ \Phi(x) \end{Bmatrix} = A_{a(n_a)} \begin{Bmatrix} \sin\frac{n_a\pi x}{L} \\ g_{a(n_a)}\cos\frac{n\pi x}{L} \end{Bmatrix} \quad (mode\ shape\ for\ \omega_{a(n_a)})$$

$$V_{a(0)}(x) = A_{a(0)} \begin{Bmatrix} 0 \\ \beta_1^o \end{Bmatrix} \quad (mode\ shape\ for\ \omega_c) \quad (38)$$

3.2.3 When $\omega > \omega_c$

Substituting eq. (21) into (23) yields the following matrix equation

$$\begin{Bmatrix} U(0) \\ \Phi'(0) \\ U(L) \\ \Phi'(L) \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & -ag_a & 0 & \bar{b}g_{\bar{b}} \\ sin(aL) & cos(aL) & sinh(\bar{b}L) & cosh(\bar{b}L) \\ -ag_a sin(aL) & -ag_a cos(aL) & \bar{b}g_{\bar{b}} sinh(\bar{b}L) & \bar{b}g_{\bar{b}} cosh(\bar{b}L) \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (39)$$

Using the same procedure as in the previous section, we have

$$A_2 = A_4 = 0 \ \ and\big(ag_a + \bar{b}g_{\bar{b}}\big)\sin(aL)\sin\big(\bar{b}L\big) = 0 \quad (40)$$

Since $(ag_a + \bar{b}g_{\bar{b}}) \neq 0$ and $\bar{b} > 0$, then if $\omega > \omega_c$ we have the following two conditions

$$\sin(a_n L) = 0 \ or \ \sin(\bar{b}L) = 0 \quad (41)$$

From which we obtain

$$a_n = \frac{n\pi}{L} \quad (n = n_a + 1, n_a + 2, \ldots, \infty) \quad (42)$$

Therefore, substituting (27) into (17) – (19) yields natural frequencies ω_n given as:

$$\omega_{a(n)} = \sqrt{Z(n) - \sqrt{Z(n)^2 - 4R(n)}}$$
$$(n = n_a + 1, n_a + 2, \ldots, \infty) \quad (43)$$

Thus, the nth mode shape corresponding to $\omega_a(n)$ from (21) is written as

$$V_{a(n)}(x) \equiv \begin{Bmatrix} U(x) \\ \Phi(x) \end{Bmatrix} = A_{a(n)} \begin{Bmatrix} \sin\frac{n\pi x}{L} \\ g_{a(n)}\cos\frac{n\pi x}{L} \end{Bmatrix} \quad (44)$$
$$(n = n_a + 1, n_a + 2, \ldots, \infty)$$

and

$$\bar{b}_m = \frac{m\pi}{L} \quad (m = 1,2, \ldots, \infty) \quad (45)$$

Therefore, substituting (27) into (17)–(19) yields natural frequencies ω_n given as:

$$\omega_{\bar{b}(m)} = \sqrt{Z(m) - \sqrt{Z(m)^2 - 4R(m)}} \quad (\omega_{\bar{b}(m)} > \omega_c)$$
(46)

Where $Z(m)$ and $R(m)$ are defined in (29) by replacing n with m Thus, the nth mode shape corresponding to $\omega_{\bar{b}(m)}$ are obtained from (21) as

$$V_{\bar{b}(m)}(x) \equiv \begin{Bmatrix} U_{\bar{b}(m)}(x) \\ \Phi_{\bar{b}(m)}(x) \end{Bmatrix} = A_{\bar{b}(m)} \begin{Bmatrix} \sin\frac{m\pi x}{L} \\ g_{b(n)}\cos\frac{m\pi x}{L} \end{Bmatrix}$$
$$(m = 1,2, \ldots, \infty)$$
(47)

where

$$g_{b(m)} = \frac{L}{m\pi\Delta_1}[\beta_1(\frac{m\pi}{L})^2 + K_1 - \omega^2] \quad (48)$$

Thus, we need to consider the following types of mode shapes for the transverse vibration of the simply supported prestressed thick beam on bi-parametric foundation.

$$V_{a(n)}(x) = A_{a(n)} \begin{Bmatrix} \sin\frac{n\pi x}{L} \\ g_{a(n)}\cos\frac{n\pi x}{L} \end{Bmatrix} \quad (mode\ shape\ for\ \omega_{a(n)})$$

$$V_{a(0)}(x) = A_{a(0)} \begin{Bmatrix} 0 \\ \beta_1^o \end{Bmatrix} \quad (pure\ shear\ mode\ shape\ for\ \omega_c)$$

$$V_{\bar{b}(m)}(x) = A_{\bar{b}(m)} \begin{Bmatrix} \sin\frac{n\pi x}{L} \\ g_{b(n)}\cos\frac{n\pi x}{L} \end{Bmatrix} \quad (mode\ shape\ for\ \omega_{b(n)})$$
(49)

3.3 The Orthogonality Conditions for the Model

In order to obtain the forced response of thick beam, we use the method of eigenfunction expansion. Therefore, the orthogonality conditions of the eigenfunctions have to be established for the beam model discussed so far. Thus, the spatial equations of the homogeneous problem (9) can be written as follow:

$$K(V_n) = \omega_n^2 M(V_n) \quad (50)$$

where V_n denotes the nth eigenfunction for the vector of $[U_n \quad \Phi_n]^T$ of the beam model and corresponds to the natural frequency ω_n^2 uniquely to within an arbitrary constant. The operators **K** and **M** are self-adjoint (with corresponding boundary conditions) if

$$\int_0^L [V_n^T K(V_m) - V_m^T K(V_n)]dx = 0$$
$$and \quad (51)$$
$$\int_0^L [V_n^T M(V_m) - V_m^T M(V_n)]dx = 0$$

Since the second condition in (51) is automatically satisfied for the model. Therefore, using equation (50), we can write the first condition in (51) as

$$(\omega_m^2 - \omega_n^2)\int_0^L V_n^T M(V_m) = 0 \quad (52)$$

However, eigenvalues are unique to the eigenfunctions, $\omega_m^2 \neq \omega_n^2$ for $(m \neq n)$. In order for the above equation to be zero, the integral has to be zero, i.e

$$\int_0^L V_n^T M(V_m) = 0 \quad (for \quad m \neq n) \tag{53}$$

This is the orthogonality condition for the eigenfunctions. When $m = n$, we normalize the eigenfunctions by setting the integral equal to one,

$$\int_0^L V_n^T M(V_m) = 1 \quad (for \quad n = 1,2,3,\dots) \tag{54}$$

Combining equations (53) and (54), we can write

$$\int_0^L V_n^T M(V_m) = \delta_{mn} \tag{55}$$

where δ_{mn} is the Kronecker delta.

For this model, the corresponding boundary condition for the self-adjoint operator **K** are found to be

$$\varphi GA\left[U_n\left(\frac{dU_m}{dx} - \Phi_m\right) - U_m\frac{dU_n}{dx} - \Phi_n\right]\Big|_o^L + \left[\Phi_n\frac{d\Phi_m}{dx} - \Phi_m\frac{d\Phi_n}{dx}\right]\Big|_o^L \tag{56}$$

Then, substituting (49) into (54), we derive the coefficients of each normal mode shape as follows:

$$A_{a(n)} = \sqrt{\frac{2}{L\left(1 + g_{a(n)}^2\right)}}; \quad A_{\bar{b}(m)} = \sqrt{\frac{2}{L\left(1 + g_{b(n)}^2\right)}}; \quad A_{a(0)} = \frac{1}{\beta_1^o\sqrt{L}} \tag{57}$$

3.4 Modal-Asymptotic Analysis of Forced Vibration of the Model

The forced vibration of (1) can be represented by using the mode summation given as

$$v(x,t) = \sum_{n=1}^{\infty} V_{a(n)}(x)y_{a(n)}(t) + \sum_{n=1}^{\infty} V_{b(n)}(x)y_{b(n)}(t) + V_{a(0)}(x)y_{a(0)}(t) \tag{58}$$

where **v**(x,t) stand for $[u(x,t) \quad \phi(x,t)]^T$ in this model while $y_{a(n)}(t)$, $y_{b(n)}(t)$, and $y_{a(0)}(t)$ are the generalized time-dependent coordinates to be determined in order to satisfy initial boundary condition. Therefore, substituting (58) into (1) and applying the orthogonality conditions of the normal mode shapes yield the following equations

$$\frac{d^2y_{a(n)}}{dt^2} + \omega_{a(n)}^2 y_{a(n)} = f_{a(n)}; \quad \frac{d^2y_{b(n)}}{dt^2} + \omega_{b(n)}^2 y_{b(n)} = f_{b(n)}; \quad \frac{d^2y_{a(0)}}{dt^2} + \omega_{a(0)}^2 y_{a(n)} = f_{a(0)} \quad (n = 1,2,3,\dots) \tag{59}$$

where the generalized forces are defined by

$$f_{a(n)} = \int_0^L V_{a(n)}(x)f(x,t)dx; \quad f_{b(n)} = \int_0^L V_{b(n)}(x)f(x,t)dx; \quad f_{a(0)} = 0 \tag{60}$$

where $f(x,t)$ is the distributed load parameter given in (2). For this problem, the distributed load moving on the beam has mass commensurable with the mass of the beam. Consequently, the load inertia is not negligible but significantly affects the behavior of the dynamical system. Thus, the distributed load $F(x,t)$ takes the form

$$F(x,t) = P_o H[x-ct]\left[1-\frac{1}{g}\left(\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial x\partial t}\right.\right.$$
$$\left.\left.+c^2\frac{\partial^2}{\partial x^2}\right)u(x,t)\right] \qquad (61)$$

where P_o is the magnitude of the transverse distributed force, g is the acceleration due to gravity, c is the initial velocity, $H[x-ct]$ is the Heaviside function which is a typical engineering function made to measure engineering applications involving functions that are either "on" of "off". Therefore, incorporating equations (60)–(61) into (59), thereafter, evaluating the integrals with the use of Fourier sine series representation of the Heaviside function, yields the following equations

$$\frac{d^2y_{a(n)}}{dt^2}+\omega_{a(n)}^2 y_{a(n)}(t)+\varepsilon_a\{[L\psi_{11}(m,k)$$
$$+\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{12}(m,n)$$
$$-\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{13}(m,n)]\ddot{y}_{a(n)}(t)$$
$$+2c[L\psi_{21}(m,n)+\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\,\pi ct}{2n+1}\psi_{22}(m,n)$$
$$-\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\,\pi ct}{2n+1}\psi_{23}(m,n)]\dot{y}_{a(n)}(t)$$
$$+c^2[L\psi_{31}(m,n)+\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\,\pi ct}{2n+1}\psi_{32}(m,n)$$
$$-\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\,\pi ct}{2n+1}\psi_{33}(m,n)]y_{a(n)}(t)\}$$
$$=P_{a(n)}[\cos\theta_n t+R_n]; \qquad (62)$$

$$\frac{d^2y_{b(n)}}{dt^2}+\omega_{b(n)}^2 y_{b(n)}(t)+\varepsilon_b\{[L\psi_{11}(m,k)$$
$$+\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{12}(m,n)$$
$$-\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{13}(m,n)]\ddot{y}_{b(n)}(t)$$
$$+2c[L\psi_{21}(m,n)+\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\,\pi ct}{2n+1}\psi_{22}(m,n)$$
$$-\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\,\pi ct}{2n+1}\psi_{23}(m,n)]\dot{y}_{b(n)}(t)$$
$$+c^2[L\psi_{31}(m,n)+\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\,\pi ct}{2n+1}\psi_{32}(m,n)$$
$$-\frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\,\pi ct}{2n+1}\psi_{33}(m,n)]y_{b(n)}(t)\}$$
$$=P_{b(n)}[\cos\theta_n t+R_n]; \qquad (63)$$

and

$$\frac{d^2y_{a(0)}}{dt^2}+\omega_{a(0)}^2 y_{a(n)}=P_c \qquad (64)$$

$$\varepsilon_a=\frac{A_{a(n)}P_o}{\rho gAL};\quad \varepsilon_b=\frac{A_{b(n)}P_o}{\rho gAL};\quad P_{a(n)}=\frac{A_{a(n)}P_o}{\rho AL};$$
$$P_{b(n)}=\frac{A_{b(n)}P_o}{\rho AL};\quad P_c=\frac{A_{a(0)}}{\rho A};\quad \theta_n=\frac{n\pi c}{L};$$
$$R_n=-(-1)^n;\quad \psi_{11}=\frac{L}{2};$$
$$\psi_{12}=\int_0^L\sin(2n+1)\sin\frac{n\pi x}{L}\sin\frac{m\pi x}{L};$$
$$\psi_{13}=\int_0^L\sin(2n+1)\sin\frac{n\pi x}{L}\sin\frac{m\pi x}{L};$$
$$\psi_{21}=\frac{n\pi}{L}\int_0^L\cos\frac{n\pi x}{L}\sin\frac{m\pi x}{L};$$
$$\psi_{22}=\frac{n\pi}{L}\int_0^L\sin(2n+1)\cos\frac{n\pi x}{L}\sin\frac{m\pi x}{L};$$
$$\psi_{23}=\frac{n\pi}{L}\int_0^L\cos(2n+1)\cos\frac{n\pi x}{L}\sin\frac{m\pi x}{L};$$
$$\psi_{31}=-(\frac{n\pi}{L})^2\psi_{11};\quad \psi_{32}=-(\frac{n\pi}{L})^2\psi_{12};$$
$$\psi_{33}=-(\frac{n\pi}{L})^2\psi_{13}$$

(65)

By considering (62) – (63), we can derive the vibration responses of the beam for cases

Case 1: When the beam is subjected to a load of negligible inertia, i.e. by setting ε_a and ε_b to zero. This is termed **moving force** problem.

Case 2: When the beam is subjected to a load of not negligible inertia, i.e. when ε_a and ε_b are greater than zero. This is termed **moving mass** problem.

However, in this study, we shall focus on the analysis of forced vibration of the model on *case 2*

3.4.1 Modal-Asymptotic Analysis of Forced Vibration of the Model when the Beam is Traversed by Moving Mass

In this section, the solution to the entire equations (62) – (64) are sought when no terms of the equation or any of the equation is neglected. However, there are have been report in the Literature in which the external transverse force $f(x,t)$ and arbitrary initial conditions were fully considered by taking into account the pure shear mode shape $\mathbf{V}_{\boldsymbol{a}(\mathbf{0})}$ to the best knowledge of the author. Thus, the equations (62) – (63) takes the form

$$\ddot{y}_{a(n)}(t) + \frac{2\varepsilon_a c Q_2(m,n)}{1+\varepsilon_a Q_1(m,n)}\dot{y}_{a(n)}(t) + \frac{\omega_{a(n)}^2 + \varepsilon_a c^2 Q_2(m,n)}{1+\varepsilon_a Q_1(m,n)} y_{a(n)}(t) = \frac{P_{a(n)}[\cos\theta_k t + R_n]}{1+\varepsilon_a Q_1(m,n)} \quad (66)$$

$$\ddot{y}_{b(n)}(t) + \frac{2\varepsilon_b c Q_2(m,n)}{1+\varepsilon_b Q_1(m,n)}\dot{y}_{b(n)}(t) + \frac{\omega_{b(n)}^2 + \varepsilon_b c^2 Q_2(m,n)}{1+\varepsilon_b Q_1(m,n)} y_{b(n)}(t) = \frac{P_{b(n)}[\cos\theta_k t + R_n]}{1+\varepsilon_b Q_1(m,n)} \quad (67)$$

where

$$Q_1(m,n) = [L\psi_{11}(m,n) + \frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{12}(m,n - \frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{13}(m,n)]$$

$$Q_2(m,n) = [L\psi_{21}(m,n) + \frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{22}(m,n - \frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{23}(m,n)]$$

$$Q_3(m,n) = [L\psi_{31}(m,n) + \frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\psi_{32}(m,n - \frac{L}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\psi_{33}(m,n)] \quad (68)$$

In order to solve the problem, an analytical approximation method called **Asymptotic** method due to **Strubble** which is often used in treating oscillatory system will be use. By this technique, one seeks the modified frequency corresponding to the frequency of the free system due to the presence of the effect of the moving mass. Following the procedures extensively discussed in [14], the homogeneous part of equations (66) – (67) are simplified to take the form:

$$\ddot{y}_{a(n)}(t) + \omega_{ma}^2 y_{a(n)} = 0 \quad (69)$$

$$\ddot{y}_{b(n)}(t) + \omega_{mb}^2 y_{b(n)} = 0 \quad (70)$$

where

$$\omega_{ma} = \omega_{a(n)}\left[1 - \frac{\epsilon_a L}{8}\left(\psi_{11}(m,n) + \frac{c^2 n^2 \pi^2 \psi_{11}(m,n)}{\omega_{a(n)}^2 L^2}\right)\right]$$

and

$$\omega_{mb} = \omega_{b(n)}[1 - \frac{\epsilon_b L}{8}(\psi_{11}(m,n) + \frac{c^2 n^2 \pi^2 \psi_{11}(m,n)}{\omega_{b(n)}^2 L^2})] \quad (71)$$

are called modified natural frequency representing the frequency of the system due to the presence of the moving mass. Thus, the entire equations (62) – (64) reduces to

$$\ddot{y}_{a(n)}(t) + \omega_{ma}^2 y_{a(n)} = W_a(t) \quad (72)$$

$$\ddot{y}_{b(n)}(t) + \omega_{mb}^2 y_{b(n)} = W_b(t) \quad (73)$$

$$\ddot{y}_{a(0)}(t) + \omega_c^2 y_{a(0)} = P_c \quad (74)$$

where

$$W_a(t) = P_{a(n)}[\cos\theta_n t + R_n];$$

$$W_b(t) = P_{b(n)}[\cos\theta_n t + R_n]; \quad (75)$$

Therefore, solving (72) – (74) using the method of Laplace transforms in conjunction with convolution theory for unknown generalized coordinates $y_{a(n)}$, $y_{b(n)}$ and $y_{a(0)}$, and then substituted the results into (58) to obtain the vibration responses as follows:

$$v(x,t) = \sum_{n=1}^{\infty} \frac{V_{a(n)}(x)}{\omega_{ma}}\left[\int_0^t W_a(t)\sin\omega_{ma}(t-\tau)d\tau + y_{a(n)}(0)\cos\omega_{ma}t + \frac{1}{\omega_{ma}}\dot{y}_{a(n)}(0)\right]$$
$$+ \sum_{n=1}^{\infty} \frac{V_{b(n)}(x)}{\omega_{mb}}\left[\int_0^t W_b(t)\sin\omega_{mb}(t-\tau)d\tau + y_{b(n)}(0)\cos\omega_{mb}t + \frac{1}{\omega_{ma}}\dot{y}_{b(n)}(0)\right]$$
$$+ \sum_{n=1}^{\infty} \frac{V_{a(0)}(x)}{\omega_c}\left[\int_0^t P_c(t)\sin\omega_c(t-\tau)d\tau + y_{a(0)}(0)\cos\omega_{ma}t + \frac{1}{\omega_c}\dot{y}_{a(0)}(0)\right] \quad (76)$$

Thus, equation (76) represent the solution to forced vibration of prestressed thick beam of length L on bi-parametric foundation subjected to moving loads using Modal-Asymptotic Analysis. It is also clearly shows that the shear mode shape $V_{a(0)}(x)$ must be considered when a thick beam is subjected to moving loads as well as to initial rotation $\phi(x,0)$ and angular velocity $\frac{\partial\phi(x,0)}{\partial t}$.

4 Numerical Investigation

In order to investigate the dynamic response of the present study, we reconsidered the uniform simply supported Timoshenko beam that was previously employed by Esmalizadeh and Ghorashi [7]. The geometric and material properties data of the beam are as follows: length L = 27.374m; area moment of inertia I = 5.71$\times 10^{-7} m^4$; cross sectional area $A = 3.3183 \times 10^{-5}$; Young's modulus $E = 2.02 \times 10^{11}\ Nm^2$; shear modulus $G = 7.7 \times 10^{10} Nm^{-2}$, mass density $\rho = 15267 kg/m^3$, and shear correction factor taken as $\varphi = 0.7$. For the analyses of forced vibrations, we assumed that the mass $M_o = 454.08kg$ and $g = 9.8ms^{-2}$. We also assumed that the beam has null initial conditions.

4.1 Model Verification

In this subsection, we aim to verify the accuracy of the present method **MAA**. Thus, the eigenfrequency and the dynamic response of the simply supported (SS) beam is computed and compared to the existing literature. Following the work of (Kim *et al.*, 2017), we compare the natural frequencies and mode shape of the present work and (Kim *et al.*, 2017) .

It is seen from Table 1 that regardless the values of axial force and foundation stiffness, the natural frequencies $\omega_{a(n)}$ and the corresponding mode shape parameters g_a of the simply supported beam on the elastic foundation in the present work is in good agreement with that reported by (Kim *et al.*, 2017), who computed the frequency parameter without considering both axial force and foundation stiffness.

Table 1: Frequency parameter of **SS** thick beam on an elastic foundation at various values of the Mode number (n)

Mode(n) at ($K_o = 0, N_o = 0$)	g_a present	$\omega_{a(n)}$ present	Kim *et al.* g_a (2017)	$\omega_{a(n)}$ (2017)
1	0.70	6.29	0.72	6.29
2	1.21	25.19	1.44	25.14
3	1.51	56.84	2.15	56.41
4	1.93	101.43	2.86	99.92
5	3.28	159.28	3.53	155.39
Mode(n) at ($K_o = 0, N_o = 400$)	g_a present	$\omega_{a(n)}$ present	Kim *et al.* g_a (2017)	$\omega_{a(n)}$ (2017)
1	0.70	5.40	0.72	6.29
2	1.21	24.35	1.44	25.14
3	1.51	56.01	2.15	56.41
4	1.93	100.61	2.86	99.92
5	3.28	158.46	3.53	155.39
Mode(n) at ($K_o = 400, N_o = 0$)	g_a present	$\omega_{a(n)}$ present	Kim *et al.* g_a (2017)	$\omega_{a(n)}$ (2017)
1	0.70	6.23	0.72	6.29
2	1.21	25.14	1.44	25.14
3	1.51	56.79	2.15	56.41
4	1.93	101.38	2.86	99.92
5	3.28	159.23	3.53	155.39

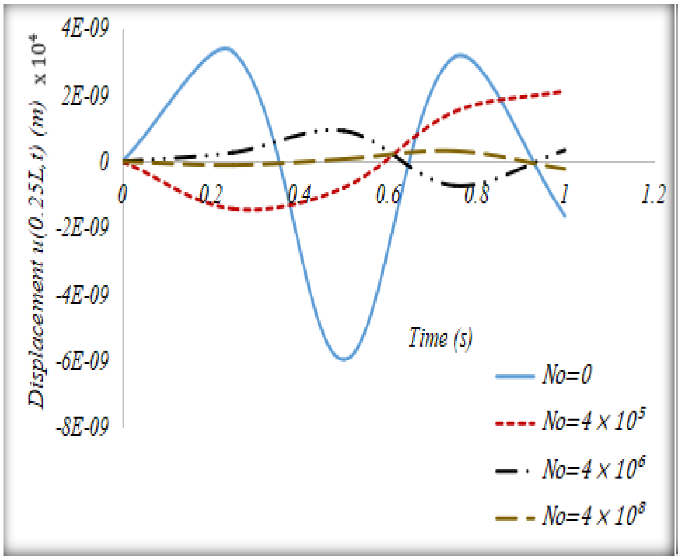

Figure 1a

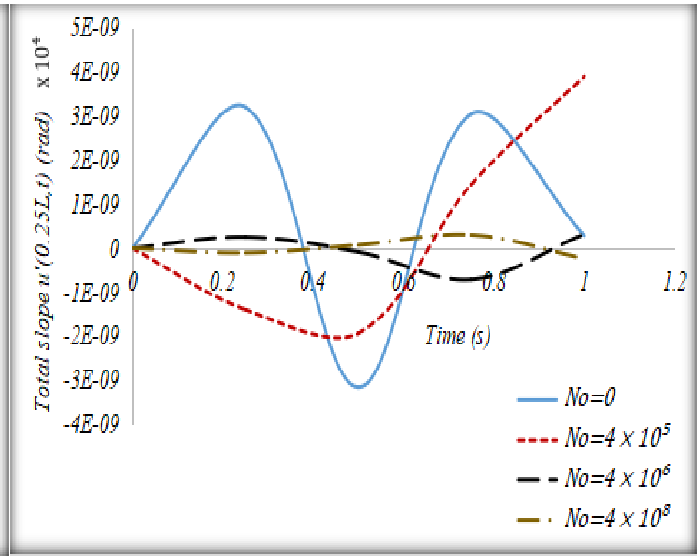

Figure 1b

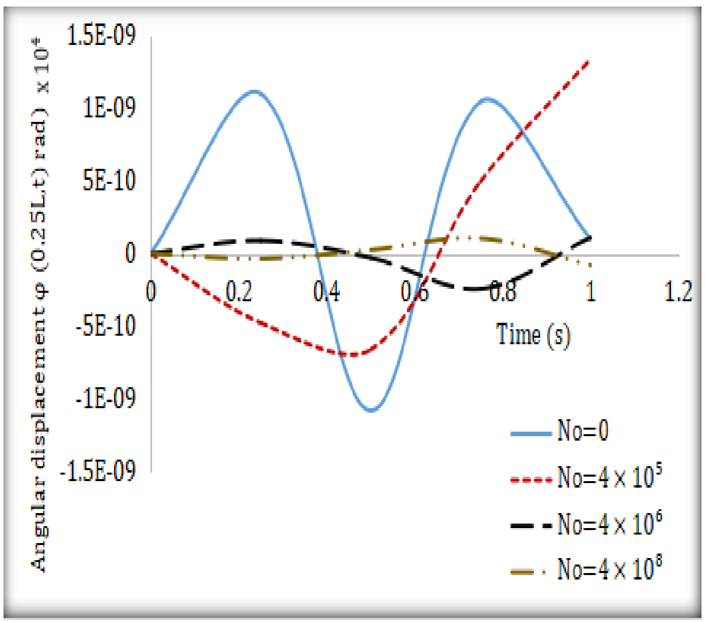

Figure 1c

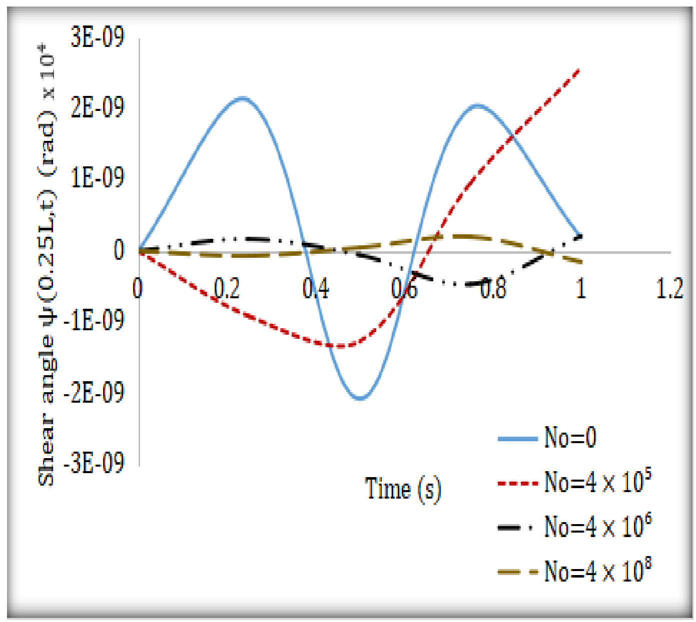

Figure 1d

Figure 1. Dynamic responses at x/L = 0.25 of a simply supported beam subjected to a distributed masses applied at x/L = 0.5 for the case of constant velocity c = 0.25vc when $K_N = 4 \times 10^4$, $Kw = 4 \times 10^4$ and excitation frequency α = 6.283: (a) transverse displacement (u); (b) total slope (u'); (c) slope due to bending (ϕ); (d) shear angle (Ψ)

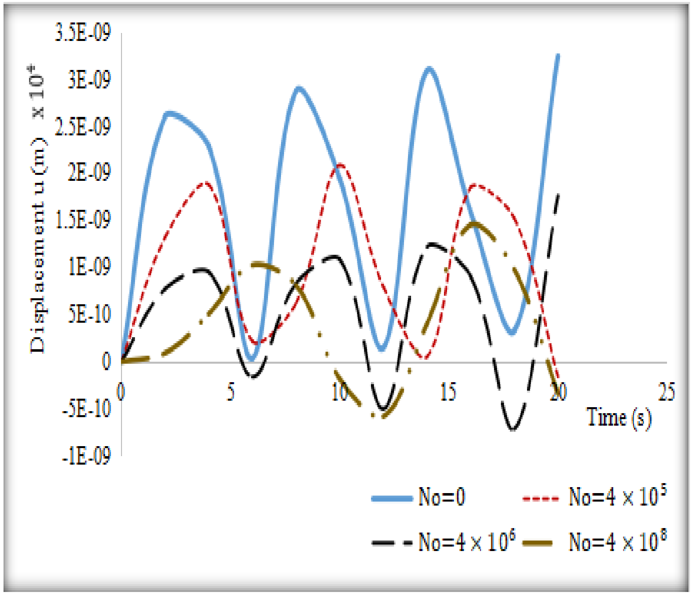

Figure 2a

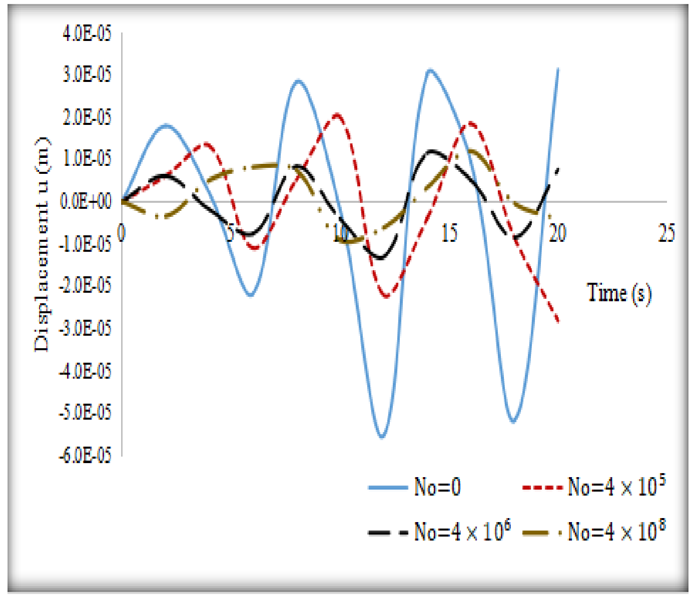

Figure 2b

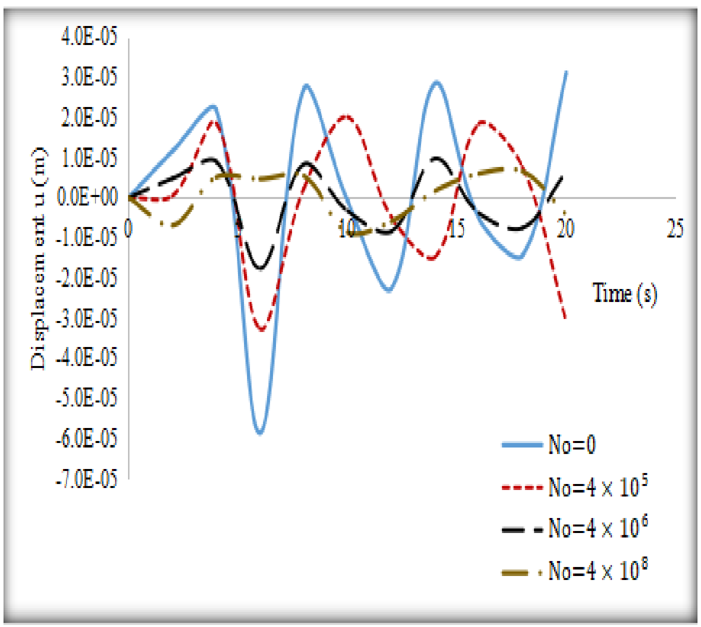

Figure 2c

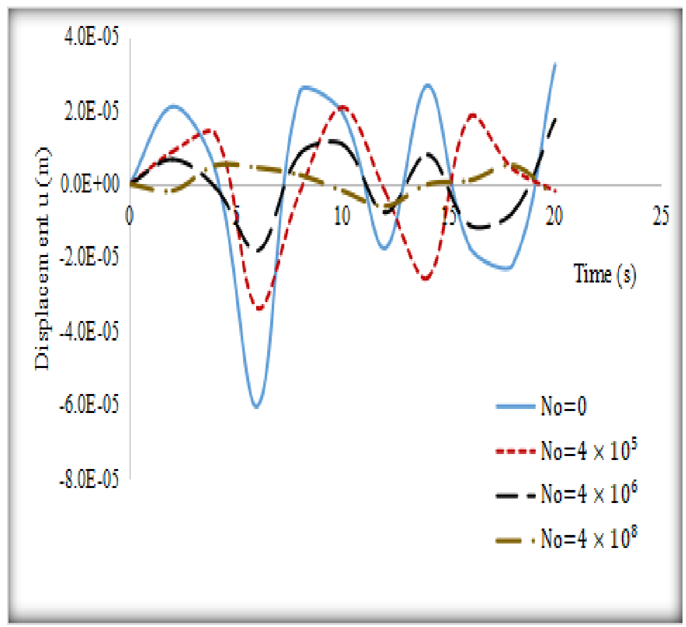

Figure 2d

Figure. 2. Effect of axial force on the dynamic response of the simply supported beam without foundation support for the case of constant velocity $c = 0.0425v_c$ and different excitation frequencies: (a) α = 0; (b) α = 40; (c) α = 80; (d) α = 120

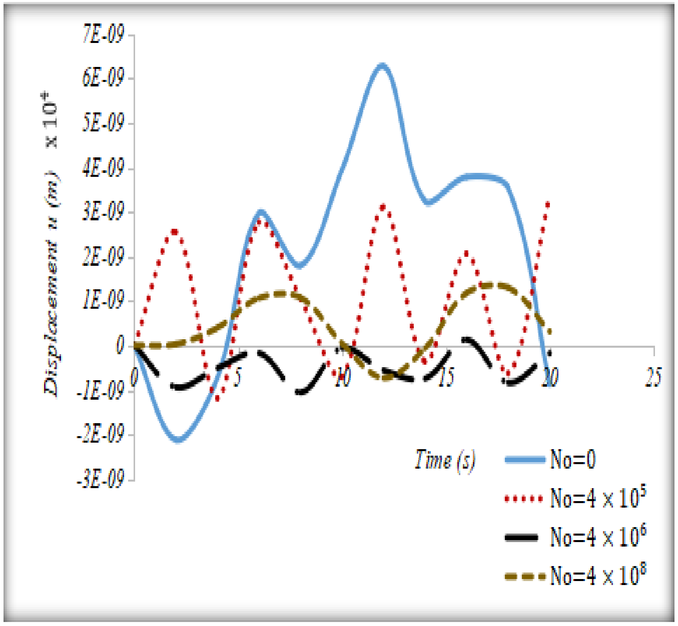

Figure 3a

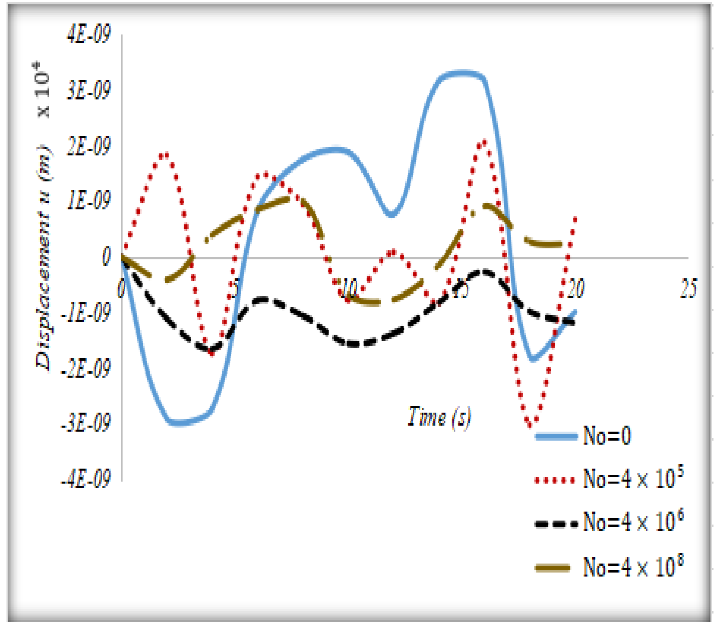

Figure 3b

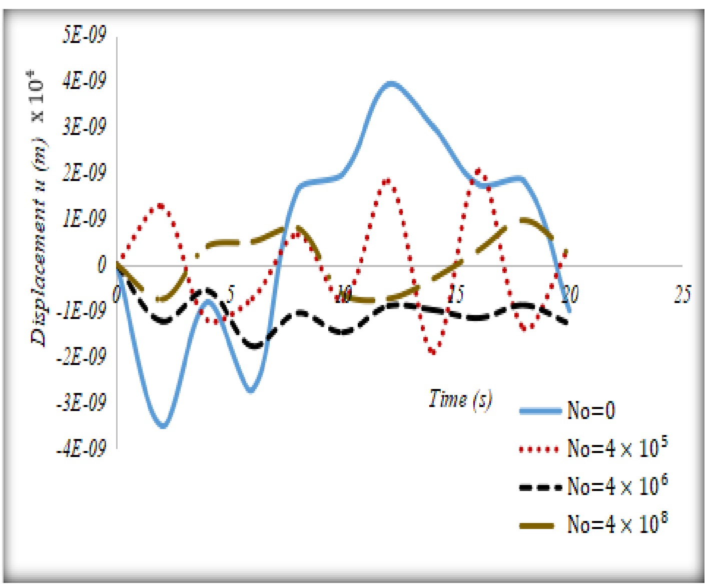

Figure 3c

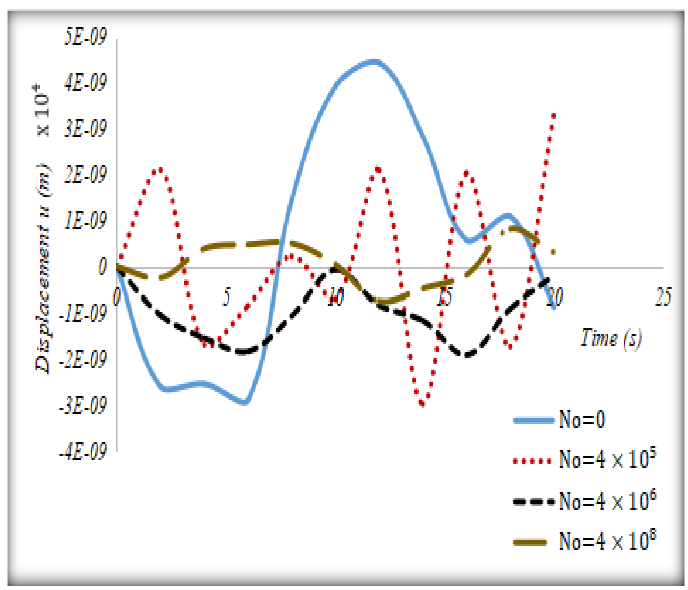

Figure 3d

Figure 3. Effect of axial force on the dynamic response of the simply supported beam with foundation stiffness $K_w = 4 \times 10^4$, and foundation modulus $Kg = 4 \times 10^4$ support for the case of constant velocity c = 0.0425vc and different excitation frequencies: (a) $\alpha = 0$; (b) $\alpha = 40$; (c) $\alpha = 80$; (d) $\alpha = 120$

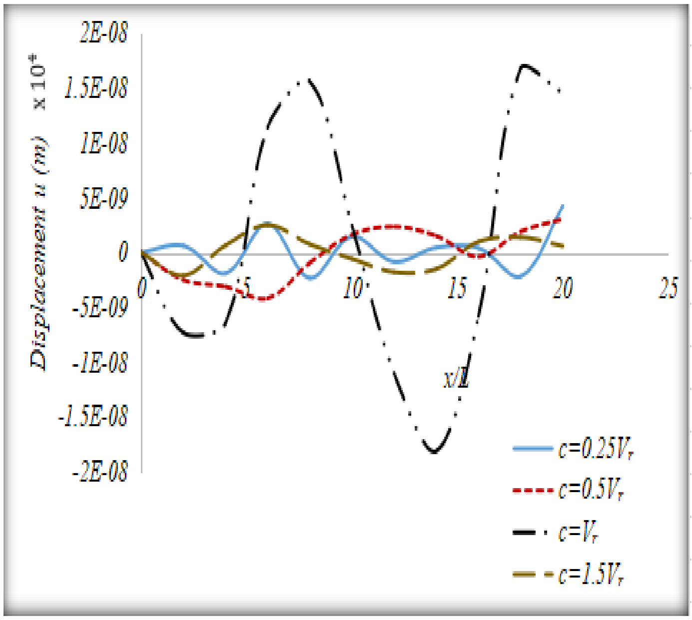

Figure 4a

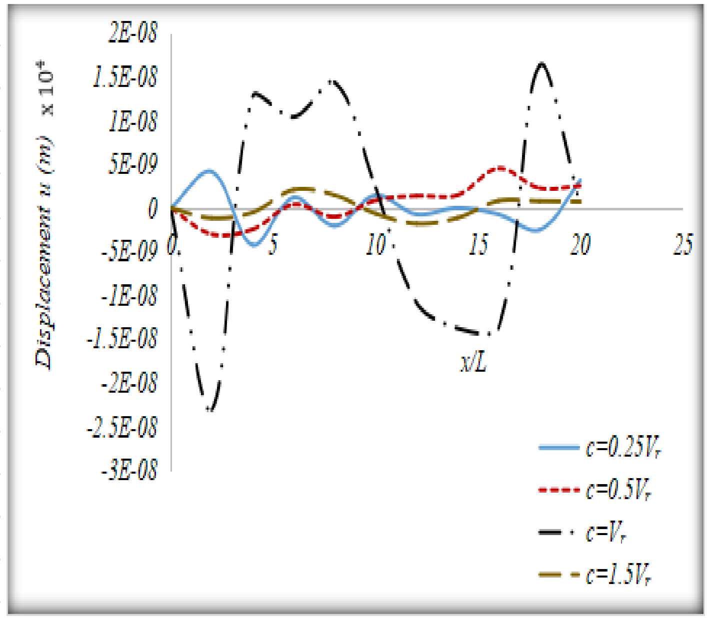

Figure 4b

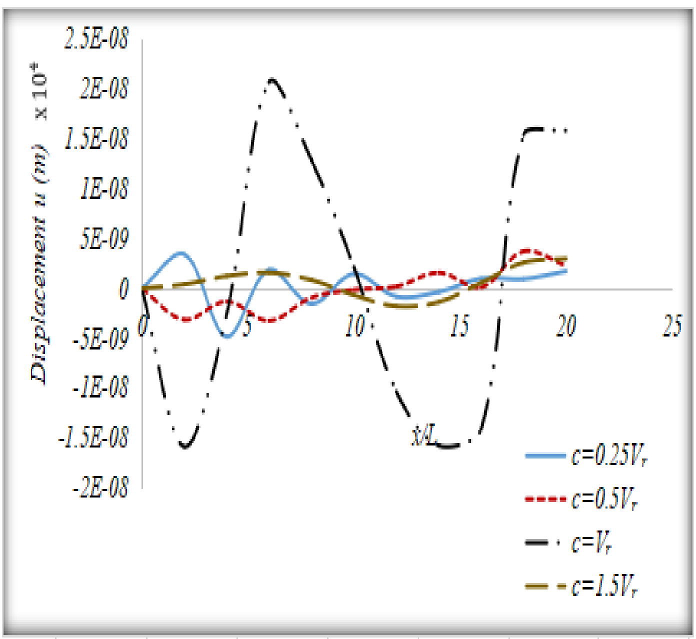

Figure 4c

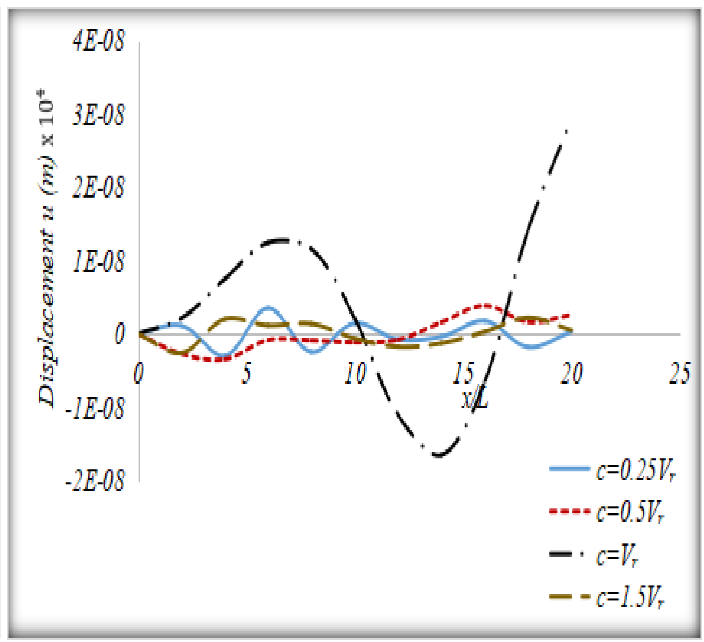

Figure 4d

Figure 4. Effect of moving velocity on the dynamic response of the prestressed simply supported beam resting on an elastic foundation with different excitation frequencies: (a) α = 20; (b) α = 40; (c) α = 60; (d) α = 80

3.2 Discussions

Figure 1 shows the dynamic responses predicted at $x/L = 0.25$, when the Timoshenko beam is subjected to a distributed force at its middle point $x/L = 0.5$. The responses are the transverse displacement $u(0.25L, t)$, the total slope $u'(0.25L, t)$, the slope due to bending $\phi(0.25L, t)$ and the shear angle due to transverse shear force $\Psi(0.25L, t) = u'(0.25L, t) - \phi(0.25L, t)$.
Figure 1 shows the deformed shape at four different values of axial force ($K_N = 0, 4\times 10^5, 4\times 10^6, 4\times 10^8$) for the dynamic responses. As seen from the figures, the dynamic deflection of the beam decreasing as the value of axial force is increasing.

The effect of axial force on the dynamic response of the simply supported Timoshenko beam without foundation support for the case of constant velocity $c = 0.0425v_c$, (where v_c is the lowest critical speed given as $v_c = 2f_1L$ and f_1 is the first natural frequency in Hz.) and different excitation frequencies is depicted in figure 2. In figures (2a) and (2b), in the case of moving load $\alpha = 0$, and in the case of moving harmonic load $\alpha = 40$ rad/s. It is noted for the two excitation frequencies that the dynamic deflection of the beams firstly decreases as the values of axial force increases but when the axial is very high say $K_N = 4 \times 10^8$, the dynamic deflection then increases while in figures (2c) and (2d) where the excitation frequencies are much higher, the dynamic deflection of continue decreases as the value of axial force increases.

In order to investigate the effect of the moving velocity on the dynamic response of the beams, the values of the axial force, foundation modulus and foundation stiffness are kept constant, say $K_N = 4 \times 10^3$, $K_g = 4\times 10^4$ and $K_w = 4\times 10^4$. The numerical computation is performed with four various values of the constant velocity, $c = 0.25v_c$, $0.5\ v_c$, v_c, $1.5\ v_c$ m/s, and at four different excitation frequencies $\alpha = 20, 40, 60, 80$ rad/s.

Figure 4 shows the effect of the moving velocity on the dynamic response of the prestressed simply supported beam resting on a one-parameter elastic foundation. As seen in figure (4a), when the excitation frequency α is 20 rad/s, the dynamic deflection of the beams firstly increases with an increment in the moving velocity, it then decreases. Regardless the value of the excitation frequency, the dynamic deflection of the beam is at the pick level when the moving velocity $c = v_c$. In order words, at a given foundation stiffness and axial force, there is a critical velocity at which the dynamic deflection reaches a maximum value for any case of excitation frequency at a given moving velocity.

4. Conclusions

The problem of the dynamic response of prestressed thick beam subjected to moving loads using modal-asymptotic analysis (MAA) has been examined. The dynamic response of the simply supported beams for moving mass case has been computed at different values of axial force, foundation stiffness, moving velocity and excitation frequency. The analyses exhibited the following features:

- The deformed shapes of thick beam strongly depend on the speeds of the moving load. There are critical speeds at which the dynamic system reaches a pick value, and this speed is called critical speed which is affected by the foundation stiffness and the excitation frequency.
- The effect of the moving velocity depends on the excitation frequency and this affect the dynamic deflection of the beam.
- A set of natural frequencies and mode shapes are presented in closed forms for frequency range $0 < \omega \leq \omega c$, where ωc is the cutoff frequency.
- As the value values of axial force parameters increases, the transverse deflection of the beam model decreased. This is strongly depending on the excitation frequency.

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